Day 24

Kalman Filter Examples

Tank of Water

- static level

plant model $$x_t = x_{t-1}$$

measurement model $$z_t = x_t + \delta_t$$

Tank of Water

- filling at a (noisy) constant rate and we do not care about the rate

plant model
$$x_t = x_{L,t-1} + \underbrace{\Delta x_L}_{u_t} + \varepsilon_t$$

measurement model
$$z_t = x_t + \delta_t$$

- u_t is the change in the water level that occurred from time t-1 to t

Tank of Water

- filling at a (noisy) constant rate and we want to estimate the rate

plant model

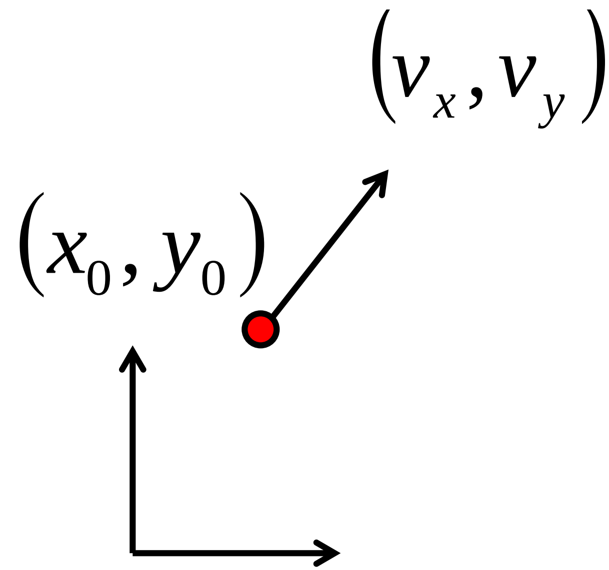
$$x_t = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{A_t} \underbrace{\begin{bmatrix} x_L \\ \Delta x_L \end{bmatrix}_{t-1}}_{x_{t-1}} + \varepsilon_t$$

measurement model

$$z_t = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_t} x_t + \delta_t$$

Projectile Motion

- projectile launched from some initial point with some initial velocity under the influence of gravity (no drag)



$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t - \tfrac{1}{2} g t^2$$

$$v_x(t) = v_x$$

$$v_y(t) = v_y - gt$$

Projectile Motion

- convert the continuous time equations to discrete recurrence relations for some time step Δt

$$
\begin{aligned}
x_t &= x_{t-1} + v_{x,t-1}\Delta t \\
y_t &= y_{t-1} + v_{y,t-1}\Delta t - \tfrac{1}{2} g \Delta t^2 \\
v_{x,t} &= v_{x,t-1} \\
v_{y,t} &= v_{y,t-1} - g\Delta t
\end{aligned}
$$

Projectile Motion

- rewrite in matrix form

$$\underbrace{\begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_t}_{x_t} = \underbrace{\begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_t} \underbrace{\begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_{t-1}}_{x_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ -\frac{1}{2} g \Delta t^2 \\ 0 \\ -gt \end{bmatrix}}_{u_t}$$

Omnidirectional Robot

- an omnidirectional robot is a robot that can move in any direction (constrained in the ground plane)
  - http://www.youtube.com/watch?v=DPz-ullMOqc
  - http://www.engadget.com/2011/07/09/curtis-boirums-robotic-car-makes-omnidirectional-dreams-come-tr/

- if we are not interested in the orientation of the robot then its state is simply its location

$$x_t = \begin{bmatrix} x \\ y \end{bmatrix}_t$$

Omnidirectional Robot

- a possible choice of motion control is simply a change in the location of the robot

$$x_t = \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}_{t-1}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{t}}_{u_t}$$

- with noisy control inputs

$$x_t = \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}_{t-1}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{t}}_{u_t} + \varepsilon_t$$

Differential Drive

- recall that we developed two motion models for a differential drive
  - using the velocity model, the control inputs are

$$u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{\alpha_1 v_t^2 + \alpha_2 \omega_t^2} \\ \varepsilon_{\alpha_3 v_t^2 + \alpha_4 \omega_t^2} \end{pmatrix}$$

Differential Drive

- using the velocity motion model the discrete time forward kinematics are

$$x_t = \begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x_c + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \\ y_c - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \\ \theta + \omega\Delta t \end{pmatrix}$$

$$= \begin{pmatrix} x - \frac{v}{\omega}\sin\theta + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \\ y + \frac{v}{\omega}\cos\theta - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \\ \theta + \omega\Delta t \end{pmatrix}$$

Eqs 5.9

Differential Drive

- there are two problems when trying to use the velocity motion model in a Kalman filter
  1. the plant model is not linear in the state and control

$$x_t = \begin{pmatrix} x - \frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t\,\Delta t) \\ y + \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t\,\Delta t) \\ \theta + \omega_t\,\Delta t \end{pmatrix}$$

  2. it is not clear how to describe the control noises as a plant covariance matrix

$$u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{\alpha_1 v_t^2 + \alpha_2 \omega_t^2} \\ \varepsilon_{\alpha_3 v_t^2 + \alpha_4 \omega_t^2} \end{pmatrix}$$

Measurement Model

- there are potentially other problems
  - any non-trivial measurement model will be non-linear in terms of the state
- consider using the known locations of landmarks in a measurement model

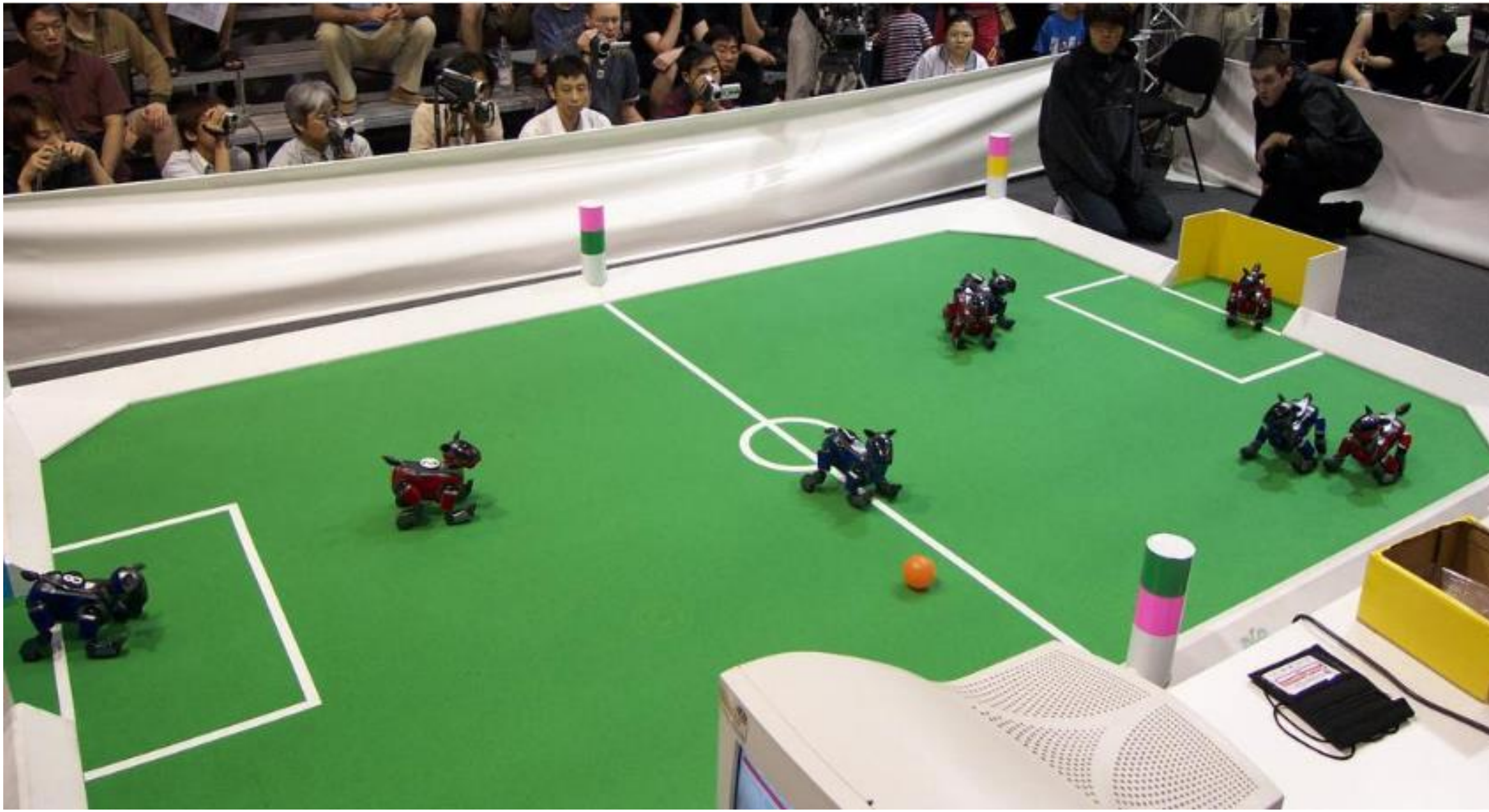
Landmarks

- a landmark is literally a prominent geographic feature of the landscape that marks a known location
- in common usage, landmarks now include any fixed easily recognizable objects
  - e.g., buildings, street intersections, monuments
- for mobile robots, a landmark is any fixed object that can be sensed

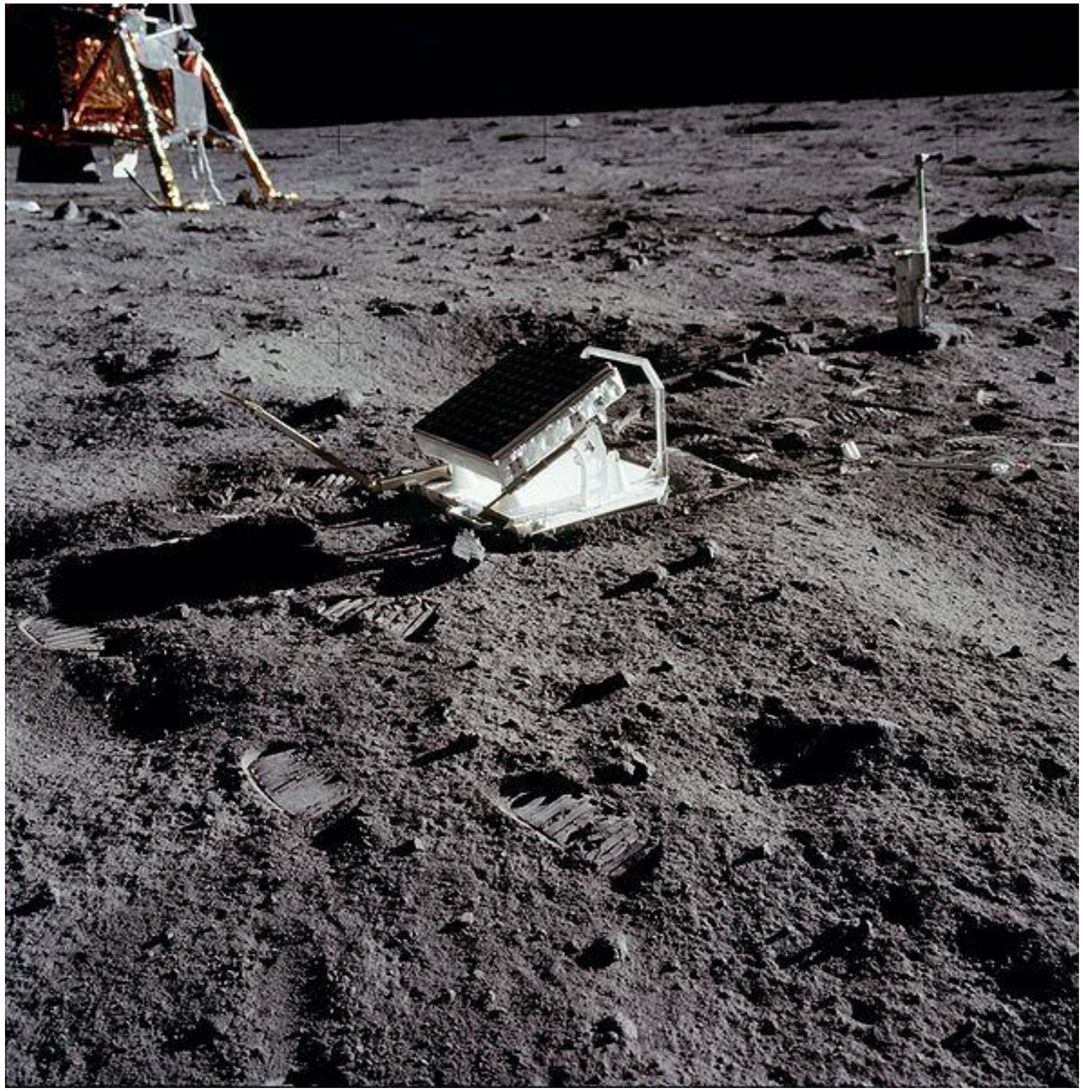
Landmarks for Mobile Robots

- visual
  - artificial or natural
- retro-reflective
- beacons
  - LORAN (Long Range Navigation): terrestrial radio; now being phased out
  - GPS: satellite radio
- acoustic
- scent?

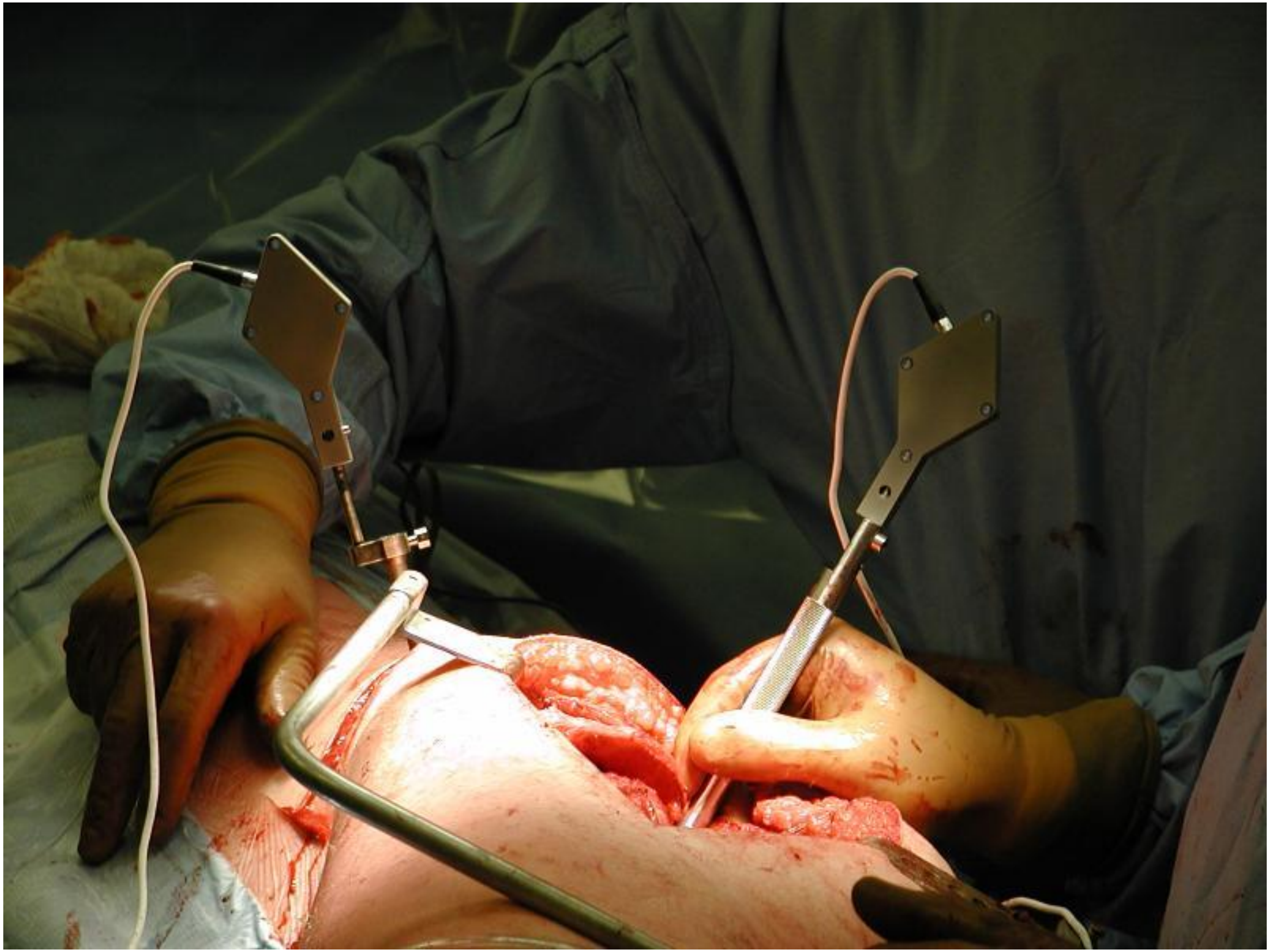
Landmarks: RoboSoccer



Landmarks: Retroreflector

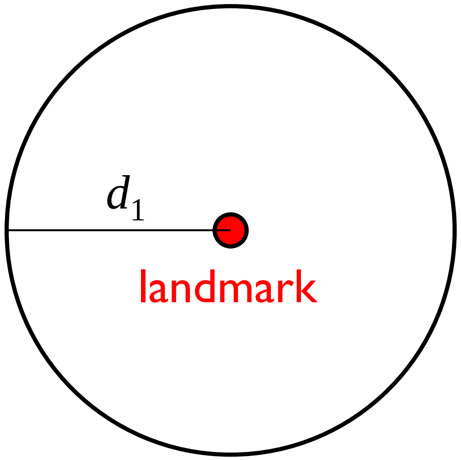


Landmarks: Active Light



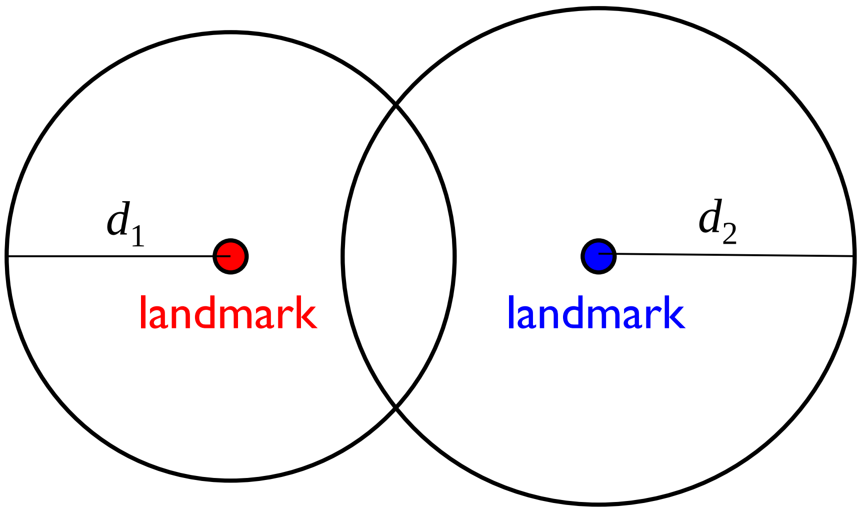
Trilateration

- uses distance measurements to two or more landmarks
- suppose a robot measures the distance d_1 to a landmark
  - the robot can be anywhere on a circle of radius d_1 around the landmark



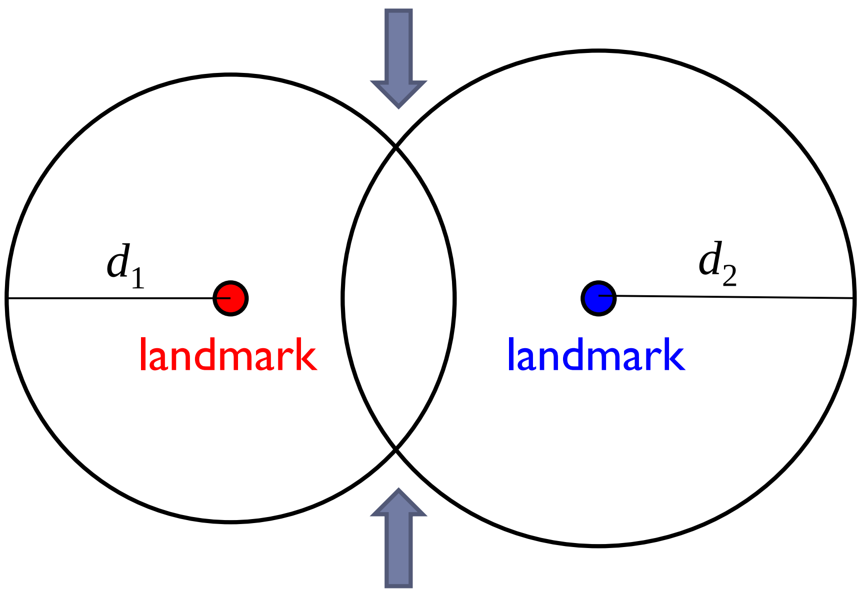
Trilateration

- without moving, suppose the robot measures the distance d_2 to a second landmark
  - the robot can be anywhere on a circle of radius d_2 around the second landmark



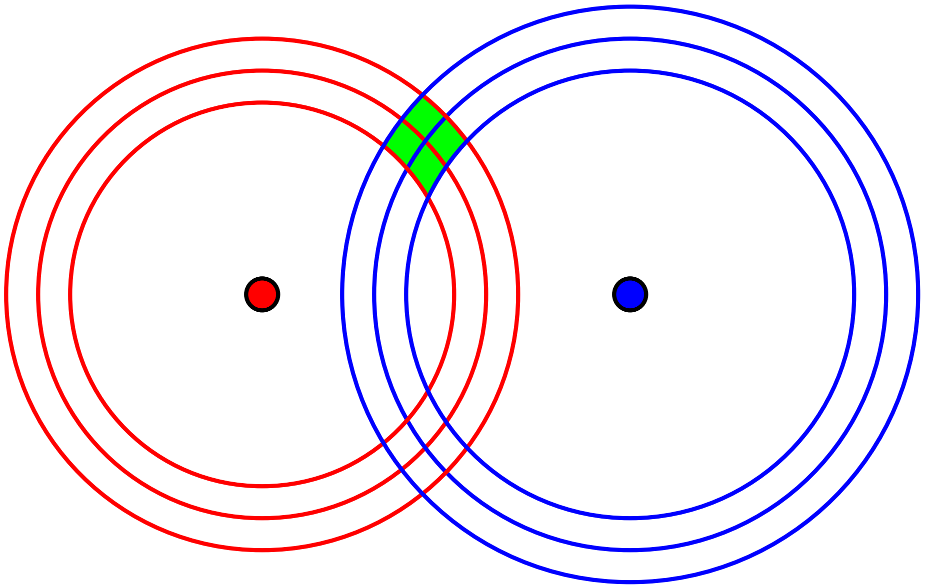
Trilateration

- the robot must be located at one of the two intersection points of the circles
  - tie can be broken if other information is known



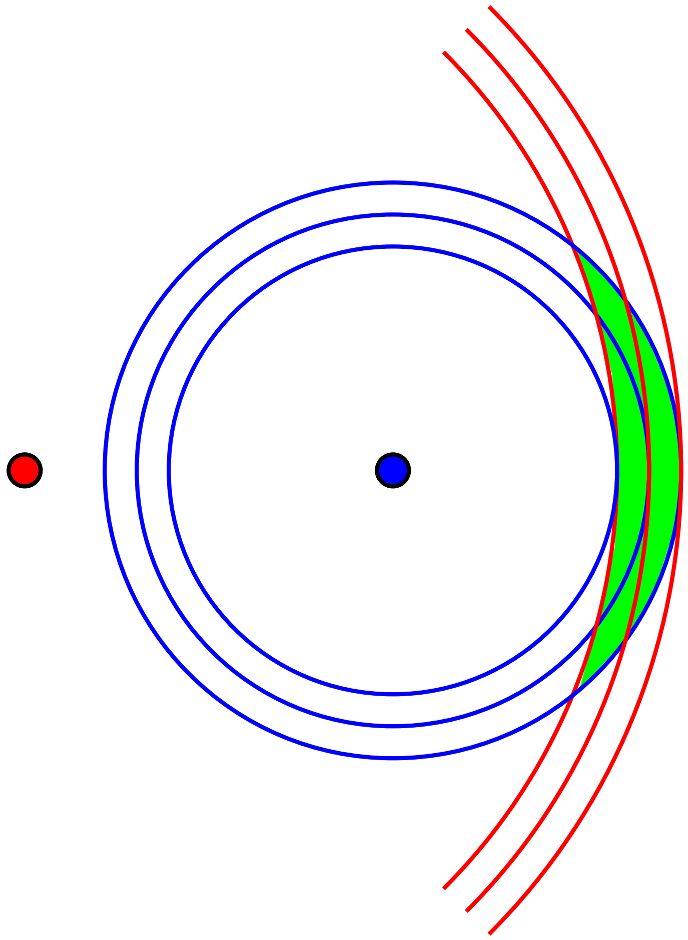
Trilateration

- if the distance measurements are noisy then there will be some uncertainty in the location of the robot



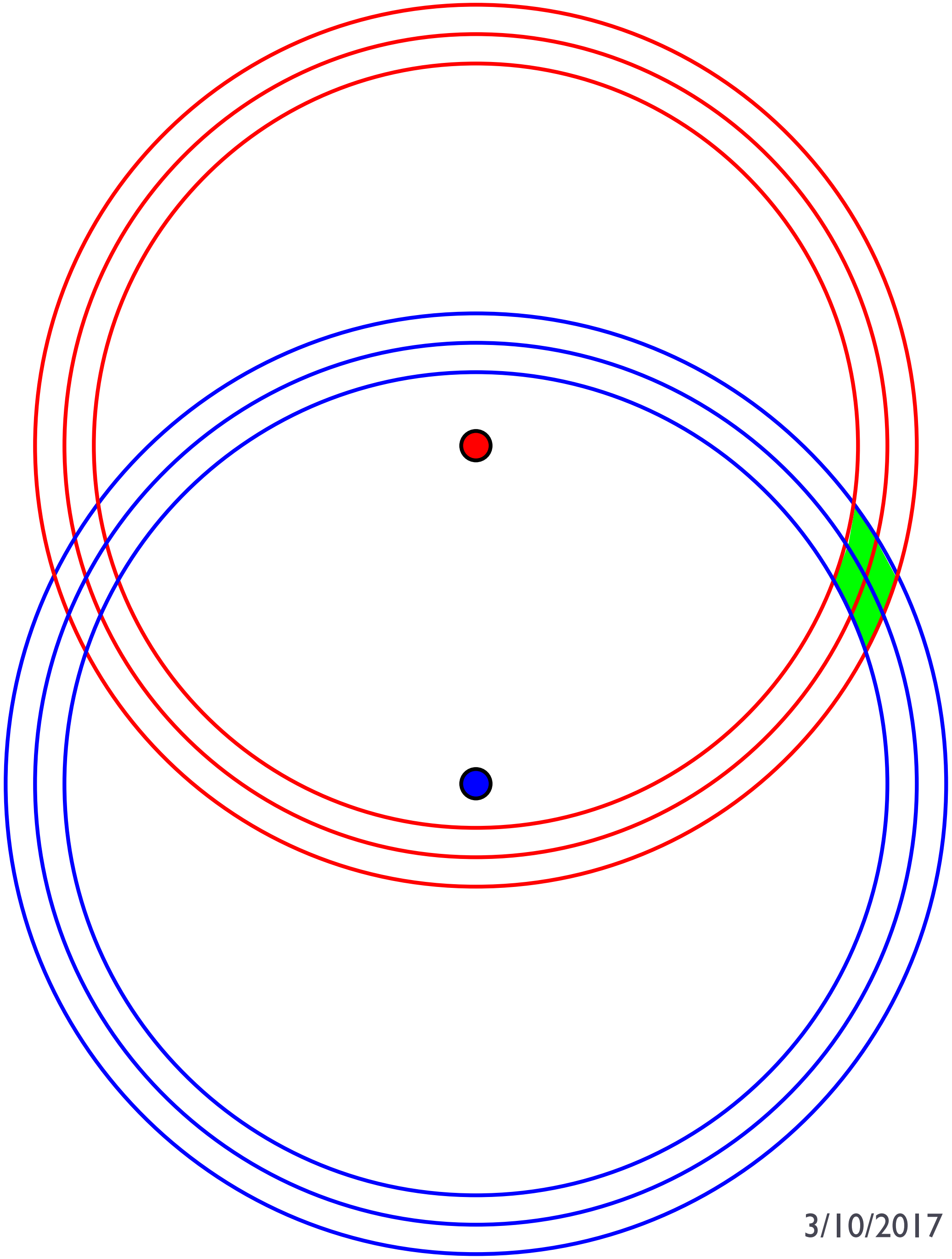
Trilateration

- notice that the uncertainty changes depending on where the robot is relative to the landmarks
- uncertainty grows quickly if the robot is in line with the landmarks



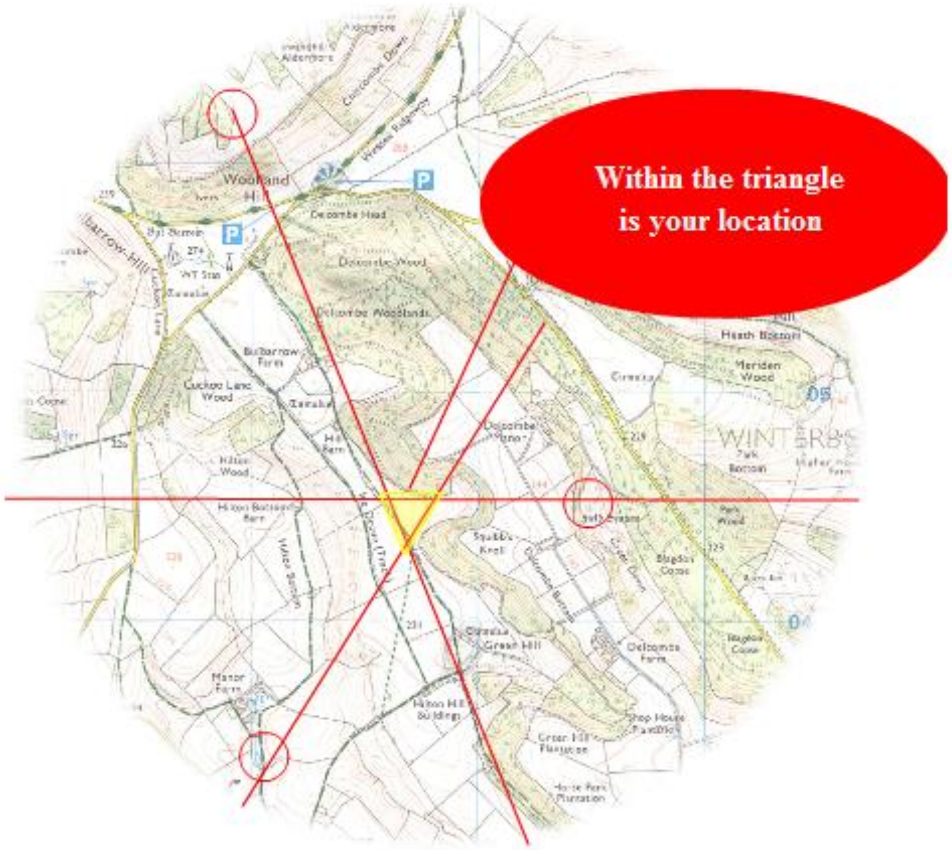
Trilateration

- uncertainty grows as the robot moves farther away from the landmarks
  - but not as dramatically as the previously slide



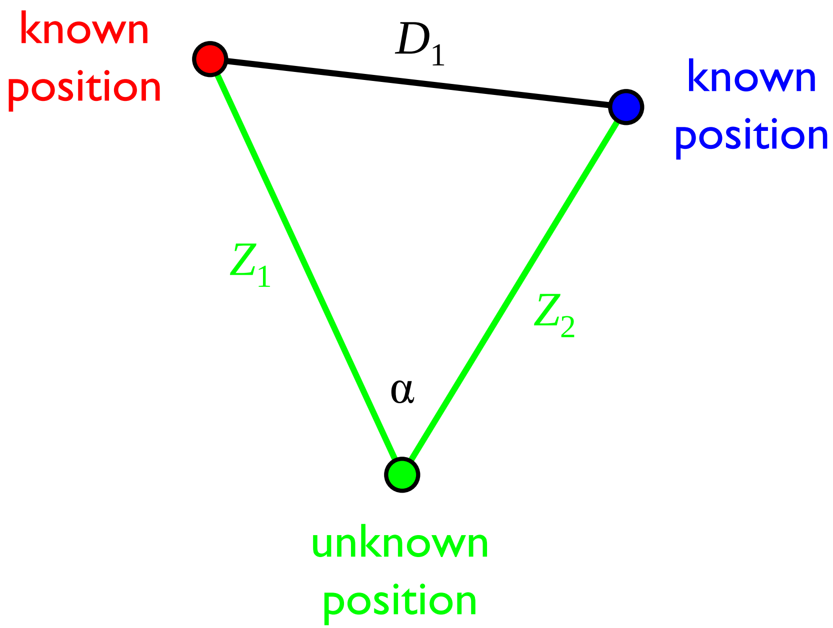
Triangulation

- triangulation uses angular information to infer position



Triangulation

- in robotics the problem often appears as something like:
  - suppose the robot has a (calibrated) camera that detects two landmarks (with known location)
    - then we can determine the angular separation, or relative bearing, α between the two landmarks



Triangulation

- the unknown position must lie somewhere on a circle arc
  - Euclid proved that any point on the shown circular arc forms an inscribed triangle with angle α
    - we need at least one more beacon to estimate the robot's location